# $CP$ violation in long baseline neutrino oscillation experiments

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## Abstract

We define a difference $D_{\rm CP}$ of the neutrino oscillation probability differences with matter effect for the CP-conjugate channels, divided by neutrino beam energy, taken between the two baselines $L = L_1$ and $L = L_2$ with $L_1/E_1 = L_2/E_2$, where $E_1$ and $E_2$ are the neutrino energy for the experiment with $L_1$ and $L_2$, respectively. The quantity $D_{\rm CP}$ doesn't contain the matter effect to the first order in $aL/2E$, $a$ representing the matter effect. We show the behavior of $D_{\rm CP}$ with $L_1 = 300$ km fixed and $L_2$ variable in the three-neutrino model.

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Where does $CP$ violation originate? In order to study the origin of $CP$ violation, we expect that the observation of $CP$ violation in neutrino oscillation experiments will be fruitful.

The neutrino oscillation is a strong means to examine the masses and mixing angles of the neutrinos [1]. The experiments have shown the solar neutrino deficit [2] and the atmospheric neutrino anomaly [3], which strongly indicate the neutrino oscillation [4]. The large mixing angle solution (LMA) by means of MSW effect [5] to the solar neutrino problem gives a mass-squared difference of $10^{-5} - 10^{-4}\mathrm{eV}^2$ [6], and the atmospheric neutrino anomaly brings the mass-squared difference of $(1.5-5)\times 10^{-3}\mathrm{eV}^2$ [7]. Especially, long baseline neutrino oscillation ecperiments are planned [8] to measure precisely the mass-squared differences and the mixing angles and, moreover, the CP violation effects in the neutrino oscillation [9]. For the long baseline experiments, however, the matter effect gives a fake CP violation effect comparable to the pure CP violation effect [10, 11]. Therefore, it is necessary to know how to distinguish the pure CP violation effect from the matter effect.

In this paper we will study the behavior of pure CP violation effects with the quantity $D_{\mathrm{CP}}$ (difference of the CP violation effects) newly introduced.

We assume three generations of neutrinos which have mass eigenstates $\nu_i^{'}$ with mass $m_i (i=1,2,3)$. The flavor eigenstates $\nu_\alpha (\alpha = e, \mu, \tau)$ and the mass eigenstates in the vacuum are related as

$$\nu_\alpha = U^{(0)}_{\alpha i} \nu_i^{'} \tag{1}$$

by mixing matrix $U^{(0)}$. We take

$$U^{(0)} = \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi c_\omega - s_\psi s_\phi c_\omega e^{i\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega e^{i\delta} & s_\psi c_\phi e^{i\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega e^{i\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega e^{i\delta} & c_\psi c_\phi e^{i\delta} \end{pmatrix} \tag{2}$$

as mixing matrix $U^{(0)}$, where $c_\psi = \cos\psi$, $s_\psi = \sin\psi$, etc.

According to Arafune, Koike and Sato's formalism [11], the evolution equation for the flavor eigenstate vector in the vacuum is expressed as

$$i\frac{d\nu}{dx} = \frac{1}{2E}U^{(0)}diag(0, \delta m_{21}^2, \delta m_{31}^2)U^{(0)\dagger}\nu \tag{3}$$

where $E$ is the energy and $\delta m_{ij}^2 = m_i^2 - m_j^2$. Similarly the evolution equation in matter is given as

$$i\frac{d\nu}{dx} = H\nu, \tag{4}$$

where

$$H \equiv \frac{1}{2E}U diag(\mu_1^2, \mu_2^2, \mu_3^2)U^\dagger. \tag{5}$$

A unitary mixing matrix $U$ and the effective mass squared $\mu_i^2 (i = 1, 2, 3)$ are determined by

$$U\begin{pmatrix} \mu_1^2 & 0 & 0 \\ 0 & \mu_2^2 & 0 \\ 0 & 0 & \mu_3^2 \end{pmatrix}U^\dagger = U^{(0)}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{pmatrix}U^{(0)\dagger} + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{6}$$

with

$$a \equiv 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5}\mathrm{eV}^2\frac{\rho}{\mathrm{gcm}^{-3}}\frac{E}{\mathrm{GeV}}, \tag{7}$$

where $n_e$ is the electron density and $\rho$ is the matter density.

The solution of Eq. (4) is

$$\nu(x) = S(x)\nu(0), \tag{8}$$

where

$$S \equiv Te^{-i\int_0^x dsH(s)} \tag{9}$$

and $T$ is the symbol for time ordering. $S$ gives the oscillation probability for $\nu_\alpha \to \nu_\beta (\alpha, \beta = e, \mu, \tau)$ at distance $L$ as

$$P(\nu_\alpha \to \nu_\beta; L) = |S_{\beta\alpha}(L)|^2. \tag{10}$$

The oscillation probability for the antineutrino $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta; L)$ is obtained by replacing $a \to -a$ and $U \to U^*$ in Eq.(10).

Taking Arafune et al.'s formalism [11] in order to calculate Eq.(10) up to the first order in $aL/2E$, we then obtain the oscillation probability $P(\nu_e \to \nu_\tau)$ in the lowest order approximation as

$$\begin{aligned}
P(\nu_e \to \nu_\tau) &= 4\sin^2\frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi^2 c_\psi^2 \left[1 - 2\frac{a}{\delta m_{31}^2}(2s_\phi^2 - 1)\right] \\
&+ 2\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E} c_\phi^2 s_\phi c_\psi \\
&\times \left[-\frac{a}{\delta m_{31}^2} s_\phi c_\psi (1 - 2s_\phi^2) + \frac{\delta m_{21}^2}{\delta m_{31}^2} s_\omega(-s_\phi c_\psi s_\omega - s_\psi c_\omega c_\delta)\right] \\
&- 4\frac{\delta m_{21}^2}{2E}\sin^2\frac{\delta m_{31}^2}{4E} c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega s_\delta, \qquad (11)
\end{aligned}$$

and $P(\nu_\mu \to \nu_e)$, $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\mu \to \nu_\tau)$ are given in Arafune et al.'s paper [11]. Recalling that $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$ is obtained from $P(\nu_\alpha \to \nu_\beta)$ by the replacement $a \to -a$ and $\delta \to -\delta$ ,we define

$$\Delta P(\nu_\alpha \to \nu_\beta) \equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta). \qquad (12)$$

Then we have

$$\begin{aligned}
\Delta P(\nu_\mu \to \nu_e) &= 16\frac{a}{\delta m_{31}^2}\left[\sin^2\frac{\delta m_{31}^2 L}{4E} - \frac{1}{4}\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E}\right] \\
&\times c_\phi^2 s_\phi^2 s_\psi^2 (1 - 2s_\phi^2) \\
&- 8\frac{\delta m_{21}^2 L}{2E}\sin^2\frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega s_\delta, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\Delta P(\nu_\mu \to \nu_\mu) &= 16\frac{a}{\delta m_{31}^2}\left[\sin^2\frac{\delta m_{31}^2 L}{4E} - \frac{1}{4}\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E}\right] \\
&\times c_\phi^2 s_\phi^2 s_\psi^2 (1 - 2c_\phi^2 s_\psi^2), \qquad (14)
\end{aligned}$$

$$\begin{aligned}
\Delta P(\nu_\mu \to \nu_\tau) &= 16\frac{a}{\delta m_{31}^2}\left[\sin^2\frac{\delta m_{31}^2 L}{4E} - \frac{1}{4}\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E}\right] \\
&\times c_\phi^2 s_\phi^2 s_\psi^2 (-2c_\phi^2 c_\psi^2) \\
&+ 8\frac{\delta m_{21}^2 L}{2E}\sin^2\frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega s_\delta, \qquad (15)
\end{aligned}$$

$$
\begin{aligned}
\Delta P(\nu_e \to \nu_\tau) &= 16\frac{a}{\delta m_{31}^2}\left[\sin^2\frac{\delta m_{31}^2 L}{4E} - \frac{1}{4}\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E}\right] \\
&\quad \times c_\phi^2 s_\phi^2 c_\psi^2 (1-2s_\phi^2) \\
&\quad - 8\frac{\delta m_{21}^2 L}{2E}\sin^2\frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega s_\delta, \qquad (16)
\end{aligned}
$$

As $\Delta P(\nu_\mu \to \nu_\mu)$ is independent of $\delta$, we see that it doesn't give the pure-$CP$ violation effect and consists only of the matter effect term.

Now we separate out the pure $CP$-violation effect from the net $CP$-violation by means of the results of experiments with two different baseline $L$ 's. Suppose that two experiments with $L = L_1$ and $L = L_2$ are available. We observe two probabilities $P(\nu_\alpha \to \nu_\beta; L_1)$ at neutrino energy $E_1$ and $P(\nu_\alpha \to \nu_\beta; L_2)$ at energy $E_2$ with $L_1/E_1 = L_2/E_2 (\alpha \neq \beta)$. Because the matter effect factor $a$ is proportional to energy $E$, we obtain the matter effect as a function of $L/E$ with dividing $\Delta P(\nu_\alpha \to \nu_\beta)$ by energy $E$ in each experiment. And we define the difference $D_{\rm CP}$ as

$$
D_{\rm CP} \equiv \left[\frac{1}{E_1}\Delta P(L_1) - \frac{1}{E_2}\Delta P(L_2)\right]_{\frac{L_1}{E_1}=\frac{L_2}{E_2}}. \qquad (17)
$$

The quantity $D_{\rm CP}$ contains no matter effect to the first order in $aL/2E$. We note that this quantity is different from the one defined by Arafune et al[11]. In Figs.1-3 we show $D_{\rm CP}$ by taking $\Delta P(L)$'s with two different baselines. In Figs.1 and 2 we show $D_{\rm CP}$ for $L_1 = 300$ km, $L_2 = 50$ km and $L_1 = 300$ km, $L_2 =$100 km, respectively. We have taken $\Delta m_{32}^2 \equiv \Delta m_{\rm atm}^2 = 2.5 \times 10^{-3}{\rm eV}^2, \Delta m_{21}^2 \equiv \Delta m_{\rm solar}^2 = 4.9 \times 10^{-5}{\rm eV}^2$, and the mixing angles and phases as $s_\omega = 0.53, s_\psi = 0.74, s_\phi = 0.16$ and $\delta = \pi/2$. Since $D_{\rm CP}$ does not involve the matter effect, we have used the exact expressions of $\Delta P(L)$ for the pure $CP$-violation effects in the computation of $D_{\rm CP}$. As can be seen in Figs.1 and 2, there are two large peaks in $D_{\rm CP}$ around $E = 0.12$ GeV and 0.2 GeV at $L = 300$km. The peaks become smaller, as the second baseline increases. In Fig.3 we compare the magnitude of $D_{\rm CP}$ for various values of $L_2$ with $L_1$ fixed as 300 km.

Finally, as the quantity $D_{\rm CP}$ does not involve the matter effect to the first order in $aL/2E$, it is not affected by the matter effect up to the order of about 5% for $\delta(D_{\rm CP})/D_{\rm CP}$ for $\rho = 3$ g/cm$^3$ and $L = 300$ km. If $\Delta P(L)$ is measured to the accuracy of 10% ($\delta(\Delta P)/\Delta P \sim 0.1$) and the neutrino beam energy is focussed to the precision of 10% ($\delta E/E \sim 0.1$), then the quantity $D_{\rm CP}$ will be observed to the accuracy of 20% ($\delta(D_{\rm CP})/D_{\rm CP} \sim 0.2$). We hope that $D_{\rm CP}$ will be measured in the future.

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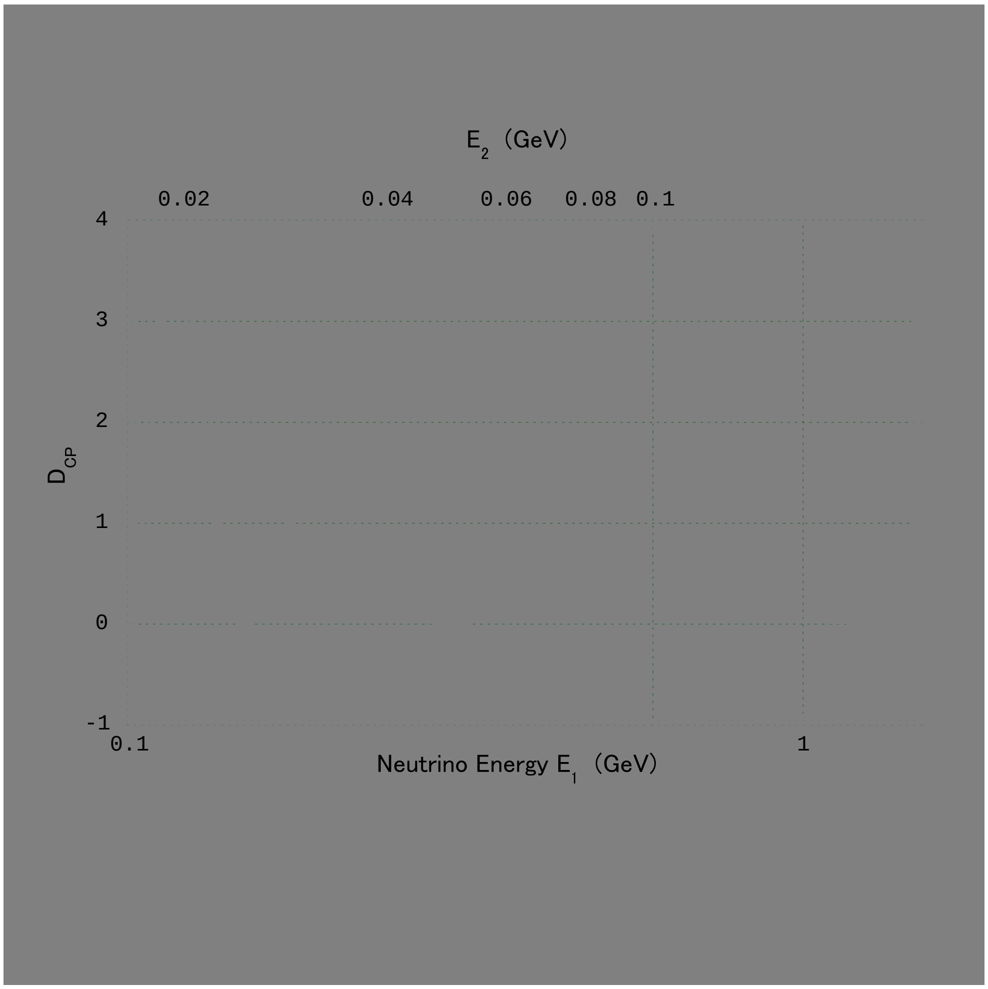


Figure 1: The difference $D_{\mathrm{CP}}$ for $L_1 = 300$km and $L_2 = 50$km. $E_1$ and $E_2$ are the neutrino energy for $L_1$ and $L_2$, respectively.

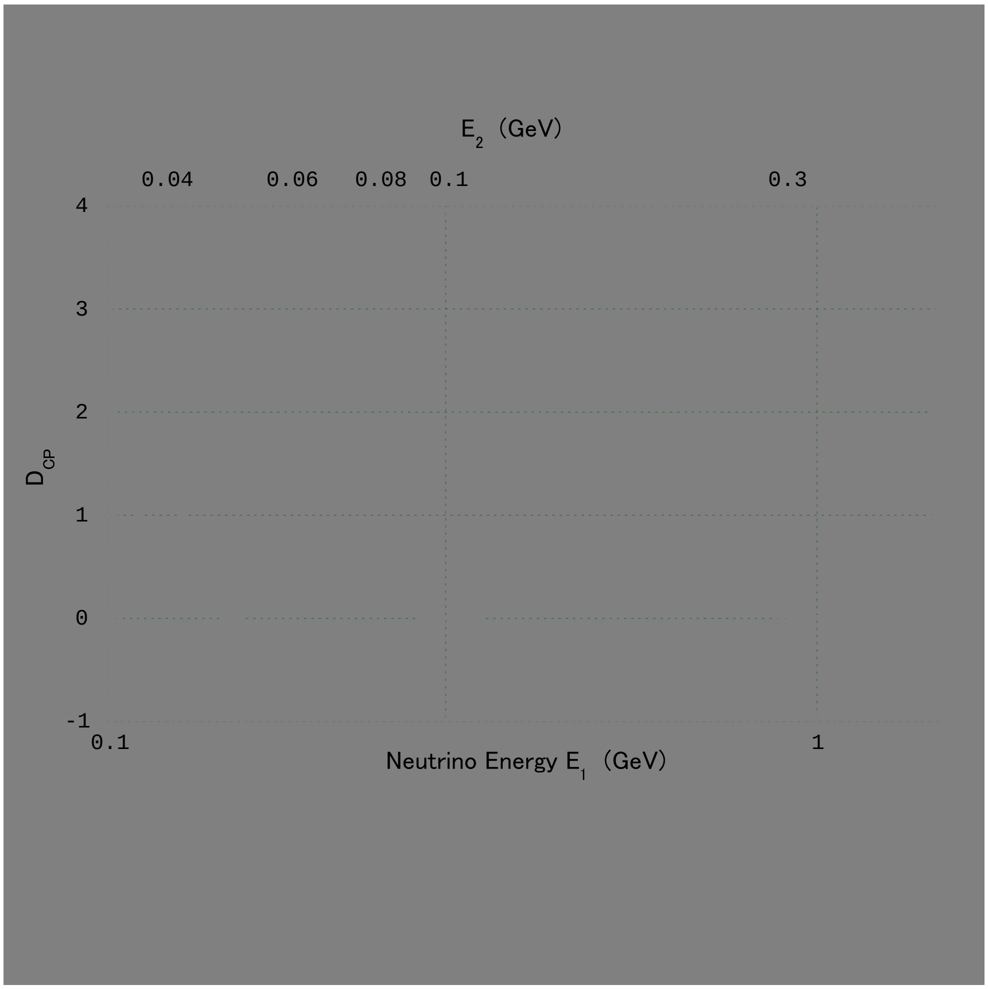


Figure 2: The difference $D_{\rm CP}$ for $L_1 = 300$km and $L_2 = 100$km. $E_1$ and $E_2$ are the neutrino energy for $L_1$ and $L_2$, respectively.

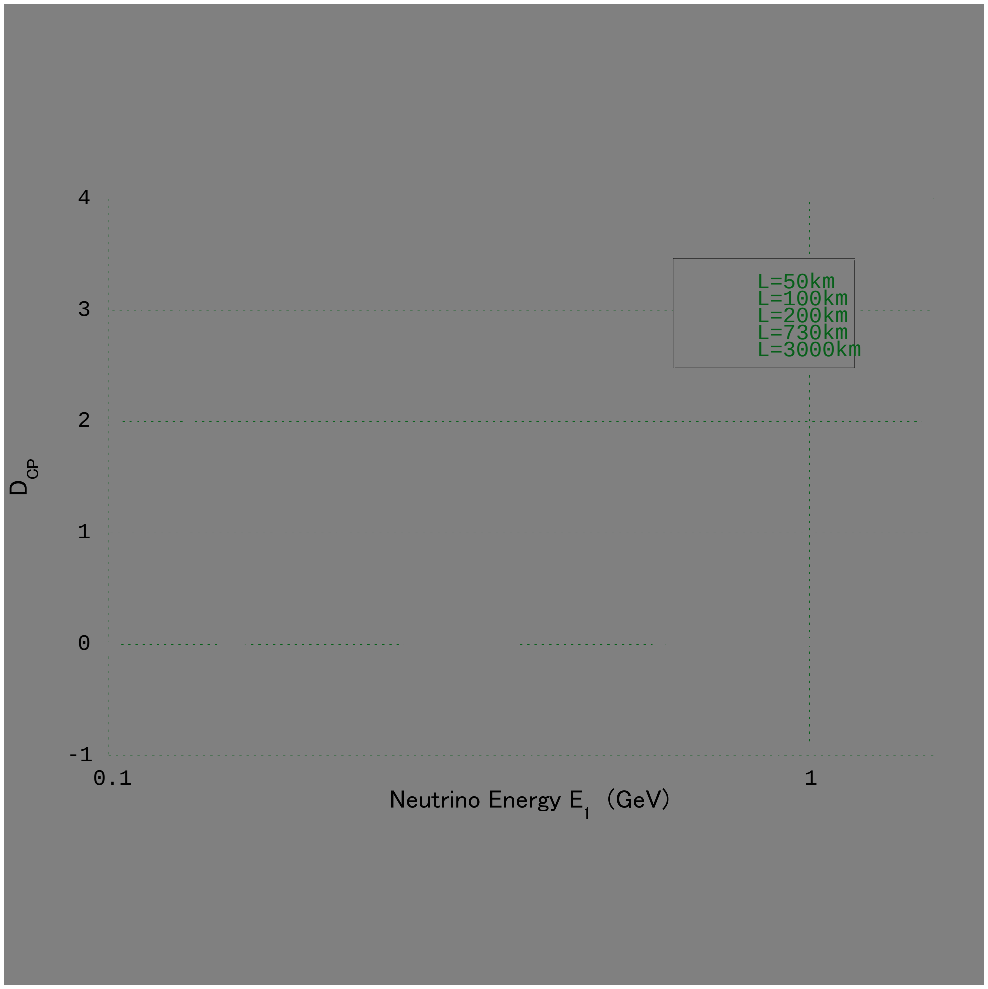


Figure 3: The difference $D_{\mathrm{CP}}$ for several values of $L_2$ with $L_1 = 300\mathrm{km}$ fixed.